Double Point Contact in Quantum Hall Line Junctions

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We show that multiple point contacts on a barrier separating two laterally coupled quantum Hall fluids induce Aharonov-Bohm (AB) oscillations in the tunneling conductance. These quantum coherence effects provide new evidence for the Luttinger liquid behavior of the edge states of quantum Hall fluids. For a two point contact, we identify coherent and incoherent regimes determined by the relative magnitude of their separation and the temperature. We analyze both regimes in the strong and weak tunneling amplitude limits as well as their temperature dependence. We find that the tunneling conductance should exhibit AB oscillations in the coherent regime, both at strong and weak tunneling amplitudes with the same period but with different functional form.

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Two-dimensional electron fluids in large magnetic fields offer an ideal setting to study nontrivial quantum coherence effects in strongly interacting macroscopic systems. It is well known that the excitations supported by the edges of quantum Hall fluids provide an ideal window to study this new physics. Already there are a number of very interesting experiments [1–5] which have uncovered the nontrivial Luttinger liquid behavior of these edge states [6,7].

Recently Kang *et al.* [8] used a new experimental setup in which two quantum Hall fluids are laterally coupled along an atomically precise barrier. In these experiments Kang and co-workers found that for filling factors $\nu \gtrsim 1$, and for some range of filling factors, a pronounced zero-bias conductance (ZBC) peak appears in the tunneling conductance of the device. (The same effect reappears for $\nu \gtrsim 2$.) Two alternative mechanisms have been proposed to explain these experiments: (i) Landau level mixing induced by the barrier potential [8–10], and (ii) tunneling at isolated quantum point contacts [11].

In Ref. [11], we showed that the salient features of the experiment of Kang *et al.* [8] can be successfully explained by modeling the system as a pair of (coupled) chiral Luttinger liquids (the edge states on each sides of the barrier) in the presence of a single point contact (PC). In particular we showed that interedge Coulomb interaction yields an effective reduced Luttinger parameter $K < 1$ and that for $\nu \gtrsim 1$, the system crosses over to the strong tunneling amplitude regime, leading to the appearance of zero-bias peak in the tunneling conductance with a peak value at $T = 0$ of $G_t = Ke^2/h$. This crossover is controlled by the energy scales of this system: the bias voltage V, the crossover scale T_K, and the temperature T (and the possibility by a small spin polarization for $\nu \sim 2$). We also predicted an increase in the height of the ZBC peak for $T \lesssim T_K$.

However, if the barrier contains more than one tunneling center (as it surely does) a number of interesting quantum coherence effects must take place, and it is of interest to investigate quantum coherence effects of multiple impurities and their competition with thermal fluctuations. An example of effects of this sort was considered some time ago by Chamon *et al.* [12], who proposed a quasiparticle interference experiment based on a two-tunneling center device in the fractional quantum Hall regime, as a way to measure directly the fractional statistics of Laughlin quasiparticles. However, in the fractional quantum Hall regime, only the case of weak tunneling centers needs to be considered since in this regime tunneling at a point contact is an irrelevant perturbation. Instead, for $\nu \gtrsim 1$, the the system is in the strong tunneling limit for $T \lesssim T_K$, which is not accessible by perturbation theory. In this regime, tunneling processes become dominant, and an instanton expansion is required to describe the physics. This problem is closely related to that of scattering centers in quantum wires, first discussed by Kane and Fisher [7,13]. Our analysis closely follows their approach.

In this Letter, we analyze the quantum Hall line junction with two-PC's both in the strong and weak tunneling amplitude limits. We show that the two-PC system may be in a coherent or in an incoherent regime depending on the distance a separating the tunneling centers. In the coherent regime the system exhibits Aharonov-Bohm (AB) oscillations in the form of a series of resonant tunneling processes in the strong tunneling limit. Instead, the AB effect in the weak tunneling limit has a simple sinusoidal form. In contrast, in the incoherent regime, the strong and weak tunneling limits are related by duality. Naturally, a realistic barrier must contain more than two-PC's, which will result in a more complex structure of AB oscillations than what we find for just two-PC's. Nevertheless it is also natural to expect that as the temperature is lowered this pattern will reveal itself step by step with the strongest PC's giving rise to the most prominent features of the interference pattern. The observation of this AB interference pattern will provide a way to sort out whether the ZBP observed by Kang *et al.* [8] is due to Landau level

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mixing or to PC tunneling, since the former mechanism predicts a smooth nonperiodic dependence of the tunneling conductance on the magnetic field.

We begin by describing our model (see Fig. 1) which has two-PC's, one located at $x = -a/2$ and the other located at $x = a/2$. In the notation of Ref. [11], the local tunneling operator is

$$\mathcal{H}_t = t_1 \psi_+^\dagger \psi_- \delta(x + a/2) + t_2 \psi_+^\dagger \psi_- \delta(x - a/2) + \text{H.c.}, \qquad (1)$$

where t_1 and t_2 are tunneling amplitudes, hereafter referred to as the "coupling constants." The right and left moving chiral Fermi fields $\psi_\pm^\dagger$ can be bosonized [14] in terms of the right and left moving chiral bosons $\phi_\pm$, as $\psi_\pm^\dagger(x) \propto (1/\sqrt{2\pi}) e^{\pm i\phi_\pm(x) \pm ik_F x}$. Similarly, the normal-ordered density operators are given by $J_\pm = -(1/2\pi)\partial_x \phi_\pm$. Notice that the tunneling operators at $x = \pm a/2$ have a relative phase of $2k_F a$, which cannot be "gauged away" by shifting the bosonic field $\phi = \phi_+ + \phi_-$ by a constant. However, the Fermi momentum is directly connected to the position of the edge or the effective width d of the barrier and magnetic field through $k_F = d/2\ell^2$ where $\ell = \sqrt{\hbar c/eB}$ is magnetic length. Hence, the relative phase $2k_F a$ is actually the AB phase $2k_F a = 2\pi\Phi/\phi_0$, where Φ is the flux in the area enclosed by tunneling edge currents and ϕ_0 is the flux quantum. We now follow Ref. [11] and introduce the rescaled boson $\varphi = \phi/\sqrt{K}$, where $K = \sqrt{(1 - g_c)/(1 + g_c)}$ is the Luttinger parameter for the interedge interaction. In imaginary time, the total Lagrangian density is

$$\mathcal{L} = \frac{1}{8\pi}\left[\frac{1}{v}(\partial_\tau \varphi)^2 + v(\partial_x \varphi)^2\right] + \sum_{\sigma=\pm} \Gamma_\sigma \cos\left[\sqrt{K}\varphi - \sigma\, \pi \frac{\Phi}{\phi_0}\right]\delta\left(x + \sigma\, \frac{a}{2}\right). \qquad (2)$$

Since the tunneling perturbation acts only at the points $x = \pm a/2$, and the free Luttinger liquid action is quadratic, we can integrate out $\varphi(x)$ and write an effective action for the tunneling center degrees of freedom $\varphi(\pm a/2, \tau)$. Let us introduce new variables $X_1 \equiv [\varphi(-a/2, \tau) + \varphi(a/2, \tau)]/\sqrt{2}$, $X_2 \equiv [\varphi(-a/2, \tau) - \varphi(a/2, \tau)]/\sqrt{2}$, and to consider for now the case when two impurities have the same strength $\Gamma_+ = \Gamma_- = \Gamma$. The effective action for $X_1(\tau)$ and $X_2(\tau)$ is

$$S_{\text{eff}} = \frac{1}{\beta}\sum_{n\neq 0}\left[\frac{|\omega_n|}{4\pi(1+e^{-|\omega_n|a})}|X_n^{(1)}|^2 + \frac{|\omega_n|}{4\pi(1-e^{-|\omega_n|a})}|X_n^{(2)}|^2\right] + 2\Gamma\int_0^{1/T} d\tau \cos\left(\sqrt{\frac{K}{2}}X_1\right)\cos\left(\sqrt{\frac{K}{2}}X_2 - \pi\frac{\Phi}{\phi_0}\right), \qquad (3)$$

where $\omega_n = 2\pi nT$ are the Matsubara frequencies, $X_n^{(i)}$ are the Fourier components of $X_i(\tau)$, and T is the temperature. Notice that $(\sqrt{K}/\pi 2\sqrt{2})X_1$ measures *the charge transferred along the barrier* and $(\sqrt{K}/\pi\sqrt{2})X_2$ measures *the charge transferred to the island* (the cross-hatched region in Fig. 1).

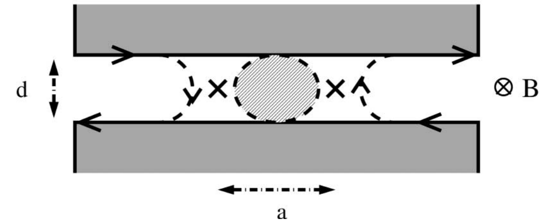

FIG. 1. A line junction with two tunneling centers. The two shaded regions represent two identical 2DEG, separated by an insulating barrier mimicking the junctions used in Ref. [8].

The term $e^{-\omega a}$ in the denominator of Eq. (3) accounts for the coherence between two-tunneling centers. As a result, at sufficiently high temperatures $Ta \gg 1$, $\exp(-\omega_n T) \ll 1$ and in this regime quantum coherence effects are washed away. Thus, in physical units, we can identify two extreme regimes in which the effective action simplifies: *the coherent regime* with $\hbar v/a \gg k_B T$ in which the PC's are strongly coupled, and *the incoherent regime* with $\hbar v/a \ll k_B T$, in which the PC's act independently. Thus, in the coherent regime, the effective action of Eq. (3) becomes

$$S_{\text{eff}} = \frac{1}{\beta}\sum_{n\neq 0}\left[\frac{|\omega_n|}{8\pi}|X_n^{(1)}|^2 + \frac{|\omega_n|}{8\pi}|X_n^{(2)}|^2\right] + \int_0^{1/T} d\tau\, V_I(X_1, X_2) + \ldots, \qquad (4)$$

where V_I is the effective potential

$$V_I(X_1, X_2) = \frac{1}{4\pi a}X_2(\tau)^2 + 2\Gamma\cos\left(\sqrt{\frac{K}{2}}X_1\right)\cos\left(\sqrt{\frac{K}{2}}X_2 - \pi\frac{\Phi}{\phi_0}\right). \qquad (5)$$

In contrast, in the incoherent regime S_{eff} reduces to

$$S_{\text{eff}} = \frac{1}{\beta}\sum_{n\neq 0}\left[\frac{|\omega_n|}{4\pi}|X_n^{(1)}|^2 + \frac{|\omega_n|}{4\pi}|X_n^{(2)}|^2\right] + 2\Gamma\int_0^{1/T} d\tau \cos\left(\sqrt{\frac{K}{2}}X_1\right)\cos\left(\sqrt{\frac{K}{2}}X_2 - k_F a\right). \qquad (6)$$

By comparing Eqs. (4) and (6) we see that in the incoherent regime the PC's are effectively decoupled while in the coherent regime they are strongly coupled, and X_2 becomes massive, with a mass of order $1/a$. Also note that the strength of nonlocal interaction in the incoherent regime is exactly twice that of the coherent regime.

(1) The coherent regime.—The potential $V_I(X_1, X_2)$ is periodic in X_1 with period $2\pi\sqrt{2/K}$. The mass term breaks this lattice translation symmetry in X_2 direction. Thus, in general there exists a single value $X_2 = X_2^0$ which minimizes the potential along the X_2 axis, unless the resonance condition $\Phi/\phi_0 =$ (half integer) is satisfied. However, when the flux satisfies $\Phi = (n + \frac{1}{2})\phi_0$, the

potential $V_I(X_1, X_2)$ acquires the additional symmetry $V_I(X_1, X_2) = V(X_1 + \pi\sqrt{2/K}, -X_2)$. Thus, in this case there are two values of X_2 which minimize the potential. This effect is analogous to the resonance phenomena first pointed out by Kane and Fisher [13] except that in that case the resonance was tuned by a gate voltage. The resonance we found here is the result of the Aharanov-Bohm effect which enables the transfer of *half* an electron when the flux penetrating the island is exactly half-integer flux quantum. We will use the instanton expansion [15–17] to examine the coherent regime in the strong tunneling limit $\Gamma \gg 1/(aK)$, and perturbation theory in the weak tunneling limit $\Gamma \ll 1/(aK)$.

The weak tunneling limit $\Gamma \ll 1/(aK)$: Here the effective potential is dominated by the mass term which is minimized for $X_2 = 0$. Hence in this case X_2 can be integrated out resulting simply in a finite flux-dependent renormalization of the tunneling amplitude: $V_I \to 2\Gamma\cos[\pi(\Phi/\phi_0)]\cos(\sqrt{K/2}X_1)$. Consequently, a lowest order perturbative calculation using the Keldysh formalism [18,19] results in an expression for $G_t(V = 0, T)$ in the coherent weak tunneling limit:

$$G_t(0, T) = K\frac{e^2}{h}\frac{K}{2}\pi^{2K}\frac{\Gamma(1/2)\Gamma(K)}{\Gamma(1/2 + K)}\left(\frac{T}{T_K^{CW}}\right)^{(2K-2)} \times \frac{1}{2}[1 + \cos(2\pi\Phi/\phi_0)] + \cdots, \quad (7)$$

where $T_K^{CW} = \Lambda(2\Gamma/\Lambda)^{1/(1-K)}$ is a crossover scale and Λ is a high-energy cutoff. Hence, in the weak tunneling limit of the coherent regime, there is an Aharonov-Bohm interference effect with the usual oscillatory form (albeit with a reduced amplitude), as well as an offset.

The strong tunneling limit: For $\Gamma \gg 1/(aK)$, the system can be either off resonance or on resonance. When the system is *off resonance*, V_I has a single minimum at $X_2 = X_2^0 \approx \sqrt{2/K}(k_F a - n\pi)$, where n is an integer. In this case, there is an energy gap of order $1/aK$ to the states with other values of X_2, i.e., the island-charge fluctuation is effectively suppressed (Coulomb blockade). Thus, X_2 is frozen to a nonzero value and the problem reduces to a single point contact system. In this regime the saturation value of the tunneling conductance is necessarily equal to Ke^2/h. We can calculate the leading corrections to this result using the instanton technique. In this case, the instanton is an electron tunneling process across the island between two disconnected pieces of the barrier. We will denote the instanton fugacity by ζ, and compute the lowest order correction to the tunneling conductance $\Delta G_t^{\text{off}} \equiv G_t - Ke^2/h$ due to these tunneling processes. To the lowest order in ζ we find

$$\Delta G_t^{\text{off}} = -\frac{\pi^{2/K}}{4}\frac{\Gamma(1/2)\Gamma(1/K)}{\Gamma(1/2 + 1/4K)}\left(\frac{T}{T_K^{\text{CSO}}}\right)^{(2/K)-2} + \cdots, \quad (8)$$

where $T_K^{\text{CSO}} = \Lambda(\frac{\Lambda}{\zeta})^{K/(1-K)}$ is a crossover scale determined by the instanton fugacity ζ and Λ.

However, when the magnetic field is tuned to a *resonance*, the projection of V_I has two degenerate minima at $X_2 = \pm\frac{\pi}{2}\sqrt{2g/K}$, where g represents the renormalization of the Luttinger parameter (or "compactification radius") of X_2; at the strong tunneling fixed point $g_0 = 1$. Hence the island effectively behaves like a two-level system and instantons connecting degenerate minima $(X_1 = m\pi\sqrt{2/K}, X_2 = X_2^0)$ and $[X_1 = (m \pm 1)\pi\sqrt{2/K}, X_2 = -X_2^0]$ with integer m correspond to electrons hopping on and off the island with the hopping amplitude ζ'. In terms of these instantons, with fugacity ζ', the partition function is [13,15]

$$Z = \sum_n \sum_{q_i^{(1)}=\pm 1}' \zeta'^{2n}\int_0^\beta d\tau_{2n}\cdots\int_0^{\tau_2} d\tau_1\, e^{-\sum_{i<j} V_{ij}}, \quad (9)$$
$$V_{ij} = -\frac{1}{2K}[q_i^{(1)}q_j^{(1)} + gq_i^{(2)}q_j^{(2)}]\ln|\Lambda(\tau_i - \tau_j)|.$$

As long as the $\zeta'/\Lambda \ll 1$, we can use this partition function to calculate semiclassically the lowest order correction to $\Delta G_t^{\text{res}} \equiv G_t^{\text{res}} - Ke^2/h$:

$$\Delta G_t^{\text{res}} = -\frac{1}{4}\pi^{(1+g)/2K}\frac{\Gamma[(1 + g)/4K]}{\Gamma[1/2 + (1 + g/4K)]} \times \left(\frac{T}{T_K^{\text{CSR}}}\right)^{[(1+g)/2K]-2}, \quad (10)$$

with $T_K^{\text{CSR}} = \Lambda(\frac{\Lambda}{\zeta'})^{4K/(1+g-4K)}$. Comparing Eqs. (8) and (10), we observe that the tunneling conductance *on resonance* is further suppressed than *off resonance*. More explicitly, we find that the *ratio* of the off and on-resonance corrections obey the scaling law

$$\frac{\Delta G_t^{\text{res}}}{\Delta G_t^{\text{off}}} \propto \left(\frac{1}{T}\right)^{(3-g)/2K}. \quad (11)$$

Thus, the Aharonov-Bohm effect leads to an oscillatory behavior of the tunneling conductance both at strong and at weak Γ. In the weak tunneling limit it leads to a small amplitude sinusoidal oscillation of G_t. At strong tunneling, although the tunneling conductance is closer to its maximum value Ke^2/h, the deviations are more pronounced and are governed by a series of resonances. In particular, although quantum coherence involves different mechanisms in the weak and strong tunneling limits, the periodicity is the same in both regimes. A simple estimate of the period is $\Delta B \approx 0.2$ Tesla for two-PC's separated by a distance $a \approx 100\ \ell$ (see Fig. 2).

Phase transition or crossover?: We now inquire if the weak and strong tunneling regimes regimes are separated by a phase transition or by a smooth crossover. The RG flow equations for this problem [13] are

$$\frac{\delta\zeta'}{\delta l} = \frac{1}{4K}[(4K - 1) - g]\zeta', \quad (12a)$$
$$\frac{\delta K}{\delta l} = -\frac{8}{\Lambda^2}\zeta'^2 g. \quad (12b)$$

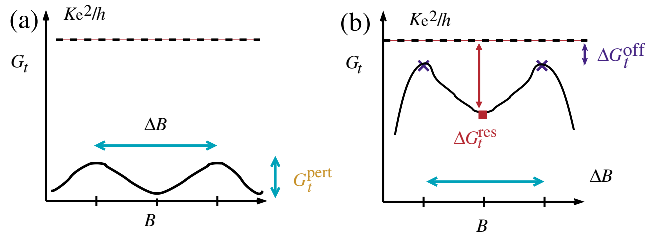

FIG. 2 (color online). (a) AB oscillations in the weak tunneling limit of the coherent regime; (b) AB effect in the strong tunneling limit of the coherent regime for $K < 1/4$.

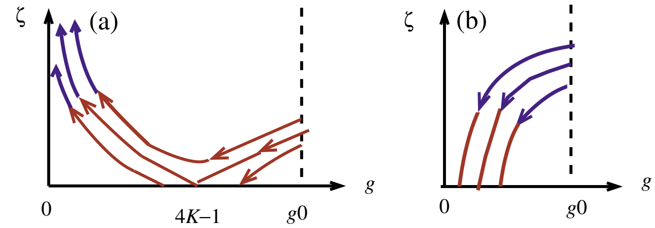

FIG. 3 (color online). RG flow for (a) $1/4 < K < 1/2$ and (b) $K < 1/4$.

This resulting flow, shown qualitatively in Fig. 3, has a fixed point at $(g, \zeta') = (4K - 1, 0)$ which depends on K, and on the initial value of g, which is $g_0 = 1$. The asymptotic behavior of the system depends on both the value of K and the initial value of ζ'. Thus, for $1/4 \leq K < 1/2$, there is a quantum phase transition at $T = 0$ between a phase in which the tunneling conductance G_t saturates to Ke^2/h as $T \to 0$, and a phase in which G_t vanishes as $T \to 0$. Instead, for $K < 1/4$, there is a crossover as the system flows to a line of strong tunneling fixed points $g \to g*$, $\zeta' \to 0$, each of which yielding a different scaling law for Eq. (11). For $T > 0$, in all cases, there will be a crossover between strong and weak tunneling fixed points as finite temperature will eventually stop the flow. The data of Ref. [8] suggests that Coulomb interactions reduces the Luttinger parameter to a small value $K \sim 0.2$.

(2) The incoherent regime.—In this regime, $\exp(-\beta a) \to 0$, thermal fluctuations overwhelm coherence effects and the two-PC's behave as if they were decoupled from each other. For $T \gg \Lambda \gg \Gamma$ we find

$$G_t = K\frac{e^2}{h} - \pi^{2/K}\frac{\Gamma(1/2)\Gamma(1/K)}{\Gamma(1/K + 1/2)}\left(\frac{T}{T_K^{\mathrm{IS}}}\right)^{(2/K-2)} + \ldots \tag{13}$$

with $T_K^{\mathrm{IS}} = \Lambda(\frac{\Lambda}{\zeta})^{K/(1-K)}$. Similarly, a semiclassical calculation in the strong tunneling limit $T \ll \Lambda$ leads to

$$G_t = \frac{e^2}{h}\frac{K\pi^{(2K)}}{2}\left(\frac{\Gamma}{\Lambda}\right)^2\frac{\Gamma(1/2)\Gamma(K)}{\Gamma(K + 1/2)}\left(\frac{T}{T_K^{\mathrm{IW}}}\right)^{(2K-2)} + \ldots \tag{14}$$

with $T_K^{\mathrm{IW}} = \Lambda(\frac{\Gamma}{\Lambda})^{1/(1-K)}$. Equations (13) and (14) show that, in the incoherent regime, weak and strong tunneling limits are exactly dual to each other. They also have the same scaling behavior as the single PC case studied in Ref. [11], which explains why the single PC picture works.

We have discussed in detail coherence effects in a two-PC system. Naturally, a realistic barrier have a number of such defects. A multipoint contact extension of our analysis leads to a complex interference pattern due to the existence of many competing pathways. Also, one expects a broad distribution of defects, both in tunneling amplitudes and in relative separation. Thus, at a given temperature, the strongest effects will be due to the closest defects with the largest tunneling amplitudes. Thus, as T is lowered, coherent Aharonov-Bohm oscillations will become increasingly more complex. Conversely, as T is raised these effects are washed out and the system will eventually reach the single impurity limit. Finally, Kataoka *et al.* [20] studied recently an antidot in a quantum Hall system for $\nu \sim 2$. We note that this device is equivalent to the strong tunneling coherent regime we discussed above, and that the temperature dependence of the oscillations they observed is remarkably similar to what we find for Aharonov-Bohm oscillations near $\nu = 1$.

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