Dirac spin-orbit torques and charge pumping at the surface of topological insulators

Papa B. Ndiaye,1,† C. A. Akosa,1,2 M. H. Fischer,3,4 A. Vaezi,5,6 E.-A. Kim,5 and A. Manchon1,7,*

1Physical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia
2RIKEN Center for Emergent Matter Science (CEMS), 2-1 Hirosawa, Wako, Saitama 351-0198, Japan
3Institute for Theoretical Physics, ETH Zurich, 8093 Zurich, Switzerland
4Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 7610001, Israel
5Department of Physics, Cornell University, Ithaca, New York 14853, USA
6Department of Physics, Stanford University, Stanford, California 94305, USA
7Computer, Electrical and Mathematical Science and Engineering Division, King Abdullah University of Science and Technology, Thuwal 23955-6900, Saudi Arabia

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We address the nature of spin-orbit torques at the magnetic surfaces of topological insulators using the linear-response theory. We find that the so-called Dirac torques in such systems possess a different symmetry compared to their Rashba counterpart, as well as a high anisotropy as a function of the magnetization direction. In particular, the damping torque vanishes when the magnetization lies in the plane of the topological-insulator surface. We also show that the Onsager reciprocal of the spin-orbit torque, the charge pumping, induces an enhanced anisotropic damping. Via a macrospin model, we numerically demonstrate that these features have important consequences in terms of magnetization switching.

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I. INTRODUCTION

Not only has spintronics yielded to the market real-deal solutions for low-energy, high-density nonvolatile memory [1], but it has also provided a fundamental understanding of the different mechanisms by which efficient electrical control of spin currents and magnetic configurations are possible. The spin-transfer-torque (STT) mechanism [2], which is central to a whole generation of memory devices, exploits the transfer of spin angular momentum between a spin-current flow and the local magnetization of a ferromagnetic (FM) layer, thereby enabling magnetization switching or precession [3]. A critical hurdle for traditional STT setups is the need for a spin-polarizer generating the spin current: STT devices comprise a number of ultrathin (anti)ferromagnetic, metallic, and insulating layers (see, e.g., Ref. [4]), rendering the design of architectures rather complex.

Research to circumvent this issue and enhance the efficiency of torque generation led to the proposal of the spin-orbit torques (SOTs) [5,6], which arise from the transfer of angular momentum between a flowing charge current and the local magnetization of a ferromagnetic layer, thereby enabling magnetization switching or precession [3]. A critical hurdle for traditional STT setups is the need for a spin-polarizer generating the spin current: STT devices comprise a number of ultrathin (anti)ferromagnetic, metallic, and insulating layers (see, e.g., Ref. [4]), rendering the design of architectures rather complex.

for spin devices made of engineered (non)magnetic materials and operated without magnetic fields [18].

The recent observation of SOTs in magnetic bilayers involving topological insulators (TIs) offers an alternative route towards efficient electrical control of the magnetization [19,20]. A three-dimensional (3D) TI is topologically distinct from a conventional 3D band insulator: it possesses an insulating bulk while hosting chiral metallic channels at the edges, where electrons are described as massless Dirac fermions with tight interlock between spin and momentum [21]. The strong spin–momentum locking results in large spin–charge conversion efficiency [22–25], as well as large SOTs enabling the control of adjacent magnetic layers [26,27]. The main strategies adopted so far consist of either doping the TI with magnetic impurities [20,28] or using the proximity effect by coating it with (possibly insulating) ferromagnets [19,29].

Various phenomena such as the topological magnetoelectric effect [30,31], STT and current-driven magnetization dynamics [32–37], the interplay between spin and charge [38–42], and spin transport in magnetic TIs [43–48] have been studied theoretically. Despite these important theoretical efforts, major puzzles remain to be understood such as the emergence of gigantic dampinglike torque [19,20], the sizable angular dependence of the SOT [20], and the significant discrepancies between the spin–charge conversion rates reported in the SOT experiments [19,20,26,27] and the spin-pumping experiments [22–25]. It is still unclear whether the spin–charge conversion efficiencies reported experimentally can be solely attributed to topological surface states [49]. It is therefore crucial to establish a solid understanding of the physics at stake at the magnetic surface of TIs in order to properly interpret these experimental results.

In this work, we explore the nature and the symmetry of nonequilibrium spin densities and their coupling to the magnetic order at TI magnetic surfaces and discuss their differences with respect to other spin-orbit-generated spin densities via the spin Hall effect [10] and Rashba effect.
[5,11]. In Sec. II, we present the model and address the electrically driven Dirac SOTs in magnetized topological insulators using the Kubo formula within the linear-response theory. We show that the effective Dirac SOT is of the form \( T = T_m(m \times E) + T_L(m \times E) \), where the in-plane and out-of-plane torques \( T_L \) exhibit a sizable anisotropy \( m \), the projection of the magnetization direction to the normal \( z \) to the TI surface and \( E \) is the applied electric field. In Sec. III, we discuss the reciprocal effect, i.e., the charge current pumped by magnetization dynamics, and show that it produces an enhanced anisotropic magnetic damping torque. Finally, in Sec. IV, we demonstrate numerically using the Landau-Lifshitz-Gilbert equation that the Dirac torque can reverse the magnetization in layers with perpendicular magnetic anisotropy but is formally less efficient than the torque arising from spin Hall effect.

II. ELECTRICALLY DRIVEN DIRAC SOTS

Let us start by considering the top surface of a three-dimensional TI in the presence of magnetic exchange, as depicted in Fig. 1. Near the Dirac point, the simplest low-energy effective Hamiltonian of the conducting surface states reads

\[
\hat{H} = \hat{H}_0 + \hat{H}_1, \tag{1}
\]

\[
\hat{H}_0 = \hbar \bar{v} \cdot (\vec{k} \times \vec{m}) + \Delta \vec{\sigma} \cdot \vec{m} - \epsilon_F, \tag{2}
\]

\[
\hat{H}_1 = \sum_i V_0 \delta(\vec{r} - \vec{r}_i), \tag{3}
\]

where \( \hat{H}_0 \) is the translationally invariant and time-independent unperturbed Hamiltonian and \( \hat{H}_1 \) accounts for random short-range disorder, treated as a perturbation in this work. In Eq. (2), the first term is the usual Rashba-type spin-orbit coupling, with \( \bar{v} \) being the Fermi velocity \((\approx 6 \times 10^5 \text{ m s}^{-1})\) in Bi\(_2\)Se\(_3\) and \( 4.3 \times 10^5 \text{ m s}^{-1} \) in Bi\(_2\)Te\(_3\)). The electron transport is confined in the \((x, y)\) plane, and \( \vec{k} = (k_x, k_y, 0) = k(\cos \phi_k, \sin \phi_k, 0) \). The second term in Eq. (2) is the exchange coupling between itinerant and local spins. Here, \( \vec{\sigma} \) is the vector of Pauli matrices, \( \Delta \) is the exchange energy, and the magnetization direction \( \vec{m} = (m_x, m_y, m_z) = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \) is uniform and can point along any (general) direction. The last term is the Fermi energy, emphasizing that we are interested in the metallic regime, away from the charge neutrality point.

The out-of-plane magnetization component is responsible for the gap opening in the TI spectrum via \( \Delta m, \vec{\varphi}_z \), thereby providing the mass of Dirac fermions. Indeed, the unperturbed Hamiltonian \( \hat{H}_0 \) can be rewritten as

\[
\hat{H}_0 = \hbar v(\vec{z} \times \vec{\varphi}) \cdot (\vec{k} + eA) + \Delta m \, \vec{\varphi}_z - \epsilon_F, \tag{4}
\]

where \( eA = \frac{F}{c} \vec{z} \times \vec{m} \) is identified as the effective vector potential [34]. Hence, the \( x \) and \( y \) components of the magnetization direction do not open a gap in the energy dispersion and only shift the Dirac cone along the \( k_{x,y} \) direction. These in-plane magnetization components are not expected to impact any physical observables as they can be straightforwardly removed by redefining the position of the Dirac node. Another particular feature of this model is that the velocity operator \( \vec{v} = \hbar \bar{v} \) is indeed directly proportional to the spin operator as \( \vec{v} = v(\vec{z} \times \vec{\varphi}) \), drawing an equivalence between the electric current \( j \) at the surface of magnetic TIs and the in-plane components of the spin density \( \vec{S} \) [37,43,46,50],

\[
j = -evz \times \vec{S}. \tag{5}
\]

This spin-velocity identity in TIs is echoed in the expressions of the response functions such as the conductivity tensor characterizing the electrical transport and the dynamical spin susceptibility. Therefore, the coefficients of the dampinglike and fieldlike torques derived below correspond to the diagonal contributions, a component aligned with the magnetization direction.

The chiral-basis eigenstates that diagonalize the unperturbed Dirac Hamiltonian \( \hat{H}_0 \) are explicitly written as

\[
|u^\pm_k \rangle = \begin{pmatrix} e^{i\gamma_k} \cos \frac{\theta_k}{2} \\ e^{i\gamma_k} \sin \frac{\theta_k}{2} \end{pmatrix}, \quad |u^\mp_k \rangle = \begin{pmatrix} -e^{i\gamma_k} \sin \frac{\theta_k}{2} \\ -e^{i\gamma_k} \cos \frac{\theta_k}{2} \end{pmatrix}, \tag{6}
\]

with

\[
\tan \gamma_k = \frac{\hbar \bar{v} \cos \phi_k - \Delta \sin \theta \cos \phi \sin \theta}{\hbar \bar{v} \sin \phi_k + \Delta \cos \phi \sin \theta}, \quad \cot \gamma_k = \frac{\Delta}{|\varepsilon_k^s|} \cos \theta,
\]

and

\[
\varepsilon_k^s = \sqrt{\hbar^2 \bar{v}^2 k^2 + \Delta^2 + 2\hbar \bar{v} \Delta \sin \theta \sin(\phi_k - \phi)}. \tag{7}
\]

The expectation value of the spin density for state \( s \) is therefore

\[
\langle S \rangle_s = \frac{\Delta}{\varepsilon_k^s} \vec{m} - \frac{\hbar \bar{v}}{\varepsilon_k^s} \vec{z} \times \vec{k}. \tag{7}
\]
two contributions [51],

\[
\delta S^I = \frac{\hbar}{2\pi V} \text{Re} \int_{-\infty}^{\infty} ds \partial_s f(s) \text{tr} \left[ \sigma \hat{G}^I_s(\Psi \cdot eE)(\hat{G}^I_s - \hat{G}^A_s) \right],
\]

(8)

\[
\delta S^I = \frac{\hbar}{2\pi V} \text{Re} \int_{-\infty}^{\infty} ds f(s) \text{tr} \left[ \sigma \hat{G}^I_s(\Psi \cdot eE) \partial_s \hat{G}^I_s - \hat{\sigma} \partial_s \hat{G}^I_s(\Psi \cdot eE) \hat{G}^I_s \right].
\]

(9)

\[\hat{G}^0_s\] are the Green’s functions defined in momentum and energy space, \(V\) is the volume of the unit cell, and \(\sigma\) accounts for the spin on the spin space as well as the summation over \(V\).

\[\delta S\] described in Refs. [46,61] leads to the renormalization of ladder diagrams through the so-called vertex correction to the proper calculation of \(\delta S\). However, in the case of any two-dimensional Dirac model such as the TI surface considered here, this second contribution vanishes in the case of any two-dimensional Dirac model such as the Fermi-sea contribution [46].

where \(\beta\) is the ultraviolet cutoff [61] in order to respect gauge invariance.

\[\hat{\sigma} = \frac{n_i V_0^2}{(\bar{\kappa} k_F)^2} [\hat{G}^0_s + \partial_s \hat{G}^I_s(\Sigma^I)]\]

For Dirac electrons, the calculation of the entire retarded self-energy in an environment with \(\delta\)-type impurities has to be done with care as logarithmic divergences naturally occur [60]. The first term of the self-energy is diagonal (with \(\delta_0\) and \(\delta_z\) components) and \(\kappa\) independent and is readily written as

\[
\hat{\Sigma}_R = -\frac{i\hbar}{4\tau} (1 + \beta m, \hat{\delta}_z),
\]

(11)

where the impurity scattering rate is given by \(\hbar/\tau = n_i V_0^2 k_F/\hbar v\) and \(\beta = \Delta/\varepsilon_F\) is the spin polarization. The second term is \(\kappa\) dependent and off-diagonal (\(= \Sigma_0 \hat{\delta}_x + \Sigma_0 \hat{\delta}_y\), and the \(\kappa\) integration should be done here with the impurity-range ultraviolet cutoff [61] in order to respect gauge invariance via the Takahashi-Ward identity [62]. The detailed procedure described in Refs. [46,61] leads to the renormalization of the velocity of the Dirac electron as \(v = (1 - \xi) v\) with \(\xi = \frac{n_i V_0^2}{24 k_F^2} \ll 1\) within the weak impurity limit and the full renormalized retarded self-energy reading now as \(\hat{\Sigma}_R = \hat{\Sigma}^R + \hat{\epsilon} \hat{k}_F \hat{\sigma} (\hat{\sigma} \times \hat{\Gamma}) \cdot \hat{\sigma} \). In the self-consistent Born approximation, the retarded Green’s function reads

\[
\hat{G}^R_k = \frac{1}{\epsilon - \hat{\epsilon}_k + i\Gamma^+(\Delta m, \hat{\delta}_z)} - \hat{\sigma} \partial_s \hat{G}^I_s(\Psi \cdot eE) \hat{G}^0_s.
\]

Notice that this form is more general than the one derived in Ref. [46], which is restricted to \(m = z\). The effective Dirac SOT is found to be of the form \(\tau = \tau m, m \times eE + \tau m \times (z \times eE)\), a form similar to the one obtained in the limit of large Rashba spin-orbit coupling in a magnetic Rashba gas [17]. The first term is odd upon magnetization reversal, proportional to the current flow (\(\sigma v\) and acts like a fieldlike torque. In contrast, the second term, \(-m, m \times eE\), is even in magnetization reversal, independent of scattering, and acts like a damping torque. While the former arises from the traditional inverse spin-galvanic effect [64], the latter is the magnetoelectric coupling identified by Garate and Franz [31]. This damping torque is quite different from the dampinglike torque stemming from the spin Hall effect, usually observed in magnetic bilayers involving heavy metals [8,13,14]. Indeed, the magnetoelectric effect at the surface of topological insulators vanishes when the magnetization lies in the plane of the surface [31], while the SHE-induced damping torque \((-m \times (z \times E) \times m)\) remains finite. Notice also that the sign of the SOT reported in Eq. (14) is opposite to the one derived for the magnetic Rashba gas [17], which is attributable to the spin chirality of the Dirac conduction band. Due to the identity between the spin and the velocity operators, Eq. (5), the fieldlike and dampinglike Dirac torque
coefficients correspond to the longitudinal and transverse (Hall) conductivities, respectively \([46,51,52]\). Finally, the SOT in Eq. (14) exhibits a complex dependence as a function of the magnetization direction, associated with the distortion of the band structure when the magnetization lies perpendicular to its magnetic direction, a precessing magnetization can pump a charge current \([65,66]\). These two effects are related to each other and considered a magnetic layer of width \(w\), thickness \(d\), and section normal to the current flow \(S = w d\).

III. CHARGE PUMPING AND ANISOTROPIC DAMPING

While a charge current can exert a torque on the local magnetization, a precessing magnetization can pump a charge current \([65,66]\). These two effects are related to each other. The charge-current density reads

\[
\mathbf{J} = \hat{\delta} \cdot \partial \mathbf{F} + \hat{g} \cdot \mathbf{E},
\]

where the electric field drives the charge current through the conductivity tensor \(\hat{g}\), while the magnetization dynamics pumps a charge current through the tensor \(\hat{\delta}\). Let us now consider a magnetic layer of width \(w\), thickness \(d\), and section normal to the current flow \(S = w d\). The particle current is defined as \(\partial n_{i} = S J_{c,i}/e\), and the electric and magnetic potentials driving the charge and magnetization dynamics read \(J_{c,i} = L_{c} E_{j}, \hat{g}_{j} = \Omega \partial \mathbf{F}\), respectively. Here, \(\Omega = L w d\) is the volume of the magnet. Therefore, Eqs. (15) and (16) can be rewritten in the more convenient form

\[
\left( \begin{array}{c} \partial n_{i} \\ \partial m_{j} \end{array} \right) = \hat{\mathcal{L}} \left( \begin{array}{c} \hat{J}_{c} \\ \hat{g}_{j} \end{array} \right),
\]

where the Onsager coefficients in \(\hat{\mathcal{L}}\) are explicitly expressed as

\[
\left( \begin{array}{cc} L_{n_{i},1} & L_{n_{i},2} \\ L_{m_{j},1} & L_{m_{j},2} \end{array} \right) = \left( \begin{array}{cc} \frac{\partial n_{i}}{\partial m_{j}} & \frac{\partial n_{i}}{\partial m_{j}} \\ \frac{\partial m_{j}}{\partial n_{i}} & \frac{\partial m_{j}}{\partial n_{i}} \end{array} \right) = \left( \begin{array}{cc} 1 & \frac{1}{\tau_{01}} \delta_{ij} \\ -\frac{1}{\tau_{01}} \delta_{ij} & 1 \end{array} \right).
\]

When applying the Onsager reciprocity principle \([66,67]\),

\[
L_{n_{i},1}(\mathbf{m}) = -L_{m_{j},1}(-\mathbf{m}),
\]

we get \(\delta_{ij}(\mathbf{m}) = -\delta_{ij}(\mathbf{m})\). In the previous section [see Eq. (14)], we showed that the torque density \(\mathbf{r}\) at the surface of the TI reads

\[
\mathbf{r} = \tau_{m} \mathbf{m} \times \mathbf{E} + \tau_{e} \mathbf{m} \times (\mathbf{z} \times \mathbf{E}),
\]

where \(\tau_{m}\) are the dampinglike and fieldlike coefficients, respectively. The total torque exerted on the ferromagnet is then

\[
\mathbf{T} = \hat{k} \cdot \mathbf{E} = \int dA \mathbf{r} (A = Lw\) is the surface area), which yields

\[
\kappa_{ij} = \frac{\mu_{B}}{M_{d}} \{ \tau_{1} m_{z}(\mathbf{m} \times \mathbf{e}_{j}) \cdot \mathbf{e}_{i} + \tau_{1} \mathbf{m} \times (\mathbf{z} \times \mathbf{e}_{j}) \cdot \mathbf{e}_{i} \}.
\]

By direct application of the Onsager reciprocity relation, we then deduce the charge-pumping coefficients in TIs, i.e.,

\[
\delta_{ij} = \frac{\mu_{B}}{M_{d}} \{ -\tau_{1} m_{z}(\mathbf{m} \times \mathbf{e}_{j}) \cdot \mathbf{e}_{i} + \tau_{1} \mathbf{m} \times (\mathbf{z} \times \mathbf{e}_{j}) \cdot \mathbf{e}_{i} \}.
\]

The charge current pumped by the magnetization dynamics simply reads

\[
\mathbf{J}_{\text{pump}} = \frac{\hbar}{2d} \{ \tau_{1} m_{z} \partial \mathbf{z} \times \partial \mathbf{m} \}.
\]

This equation establishes the correspondence between the current-driven Dirac SOT and the charge current pumped by a time-varying magnetization. By virtue of Onsager reciprocity, the results and conclusions drawn above for the SOT apply straightforwardly to the charge pumping through Eq. (22), in particular the second component, \(\sim \partial \mathbf{m}\), dominates in the metallic regime since \(\tau_{1} > \tau_{2}\). Notice that \(\mathbf{J}_{\text{pump}}\) is the current density flowing in the magnetic volume and is therefore inversely proportional to the thickness \(d\).

The charge current pumped at the surface of the TI, \(d\mathbf{J}_{\text{pump}}\), also induces an interfacial nonequilibrium spin density \(\delta S_{\text{pump}} = (d/ev)\mathbf{z} \times \mathbf{J}_{\text{pump}}\) [see Eq. (5)]. In turn, this pumped interfacial spin density induces a torque, \(\mathbf{T}_{\text{pump}} = (2\Delta/\hbar) \int dA \mathbf{m} \times \delta S_{\text{pump}}\), that reads

\[
\mathbf{T}_{\text{pump}} = \frac{\mu_{B}}{M_{d} d} (\Delta/ev) \mathbf{z} \times \mathbf{J}_{\text{pump}}
\]

\[
+ \frac{\mu_{B}}{M_{d} d} (\Delta/ev) \tau_{1} m_{z} \mathbf{z} \times (\mathbf{m} \times \mathbf{z}).
\]

The first term is odd upon time-reversal operation \((\partial_{1} \rightarrow -\partial_{1}, \mathbf{m} \rightarrow -\mathbf{m})\), while the second term is even. Accordingly, the first term contributes to the magnetic damping, while the second term renormalizes the gyromagnetic ratio. In particular, the damping torque acts only on the in-plane components of the magnetization \((m_{x}, m_{y})\), thereby creating an anisotropic damping. The total magnetic damping then reads

\[
\mathbf{T}_{\text{damping}} = -\left( \alpha + \frac{\mu_{B}}{M_{d} d} (\Delta/ev) \tau_{1} \right) \partial \mathbf{m}_{z} \mathbf{x} \times \partial \mathbf{m}_{y} + \alpha \partial \mathbf{m}_{z} \mathbf{z}.
\]

This anisotropic magnetic relaxation echoes the famous D’yakonov-Perel’ spin relaxation emerging in two-dimensional electron gases \([68]\). In recent experimental reports \([19,20,27]\), the electrical torque efficiency in TIs ranges from \((\mu_{B}/\gamma M_{d})\tau_{1} \approx 10^{-9}\) T m V\(^{-1}\) \([19]\) to \(10^{-7}\) T m V\(^{-1}\) \([20,27]\), depending on the temperature and thickness of the ferromagnet. Hence, adopting standard materials parameters \((\hbar \approx 4\ eV \ A, \Delta \approx 1\ eV)\), we obtain a damping enhancement of \((\mu_{B}/M_{d}) (\Delta/ev) \tau_{1} \approx 3 \times 10^{-4}\) to \(3 \times 10^{-2}\), which is experimentally measurable. For the sake of comparison, the enhanced damping observed in Bi\(_{2}\)Se\(_{3}/\)CoFeB bilayers lies between \(-0.03\) and \(-0.12\), with wide variability from sample to sample \([24]\).
IV. MAGNETIZATION SWITCHING BY DIRAC SOT

We conclude this study by analyzing the impact of the Dirac dampinglike torque on the magnetization reversal. In particular, we are interested in comparing the ability of the dampinglike Dirac SOT ($-m \times E$) with the dampinglike SHE-induced SOT [8] ($\sim m \times [(z \times E) \times m]$) to switch the magnetization direction of a perpendicularly magnetized FM. We study the dynamics of the magnetization within the standard macrospin approximation and numerically solve the LLG equation of motion supplemented by SOT,

$$\dot{\mathbf{m}} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \dot{\mathbf{m}} + \mathbf{T}_{\text{dirac/she}}.$$  \hspace{1cm} (25)

where $\mathbf{H}_{\text{eff}}$ is the effective field incorporating the demagnetizing field and/or an external applied magnetic field, while the last term, $\mathbf{T}_{\text{dirac/she}}$, represents the (Dirac or SHE-induced) dampinglike SOT. In the configuration we adopt, the current is driven along $x$, and the magnetic anisotropy is along $z$. The Dirac dampinglike SOT is therefore $T_{\text{dirac}} = \gamma H_{\text{dir}} m \times x$, while the SHE-induced SOT is $T_{\text{she}} = \gamma H_{\text{she}} m \times (y \times m)$, with $H_{\text{dir/she}}$ being the strength of the torque. Solving the LLG equation [Eq. (25)] while varying both the in-plane applied magnetic field $H_x$ and the SOT strength, one obtains the switching phase diagram of the macrospin as displayed in Fig. 2. Notice that Fig. 2(b) has been calculated previously [8] and is reproduced here only for comparison.

Both diagrams display the same general shape: a central diamondlike region (green) denotes the bistable state where both $+z$ and $-z$ states are stable. This region is surrounded by four regions of monostable states (blue or red), where only the $+z$ or $-z$ state is stable. Besides these general features, we observe two major differences. First, the horizontal extension of the central diamond is two times larger in the case of Dirac SOT than for SHE-induced SOT, which means that the transition between the blue and red regions is much more abrupt, which is related to the vanishing of the Dirac dampinglike SOT when the magnetization lies in the plane of the surface.

![FIG. 2. Calculated switching phase diagram with an applied in-plane field along $x$ for a current-induced (a) Dirac torque and (b) spin Hall torque (retrieve results from Ref. [8])](image)

V. CONCLUSION

To summarize, we have analytically derived the electrically driven SOTs and charge pumping at the magnetic surface of a TI. While the fieldlike Dirac torque has the same geometrical form as the standard fieldlike Rashba torque, the dampinglike Dirac torque presents a remarkable difference compared to the SHE-induced torque and vanishes when the magnetization lies in the plane of the surface. Furthermore, we uncover a strong angular dependence of the torque due to (i) the distortion of the band structure associated with the gap opening when the magnetization lies out of plane and (ii) the presence of anisotropic spin relaxation.

We note that a strong but opposite angular dependence of the torque has been experimentally reported in magnetically doped topological insulators by Fan et al. [20]: in this experiment the magnitude of the torque is larger when the magnetization lies perpendicular to the plane of the surface. Another difference between Eq. (14) and the experimental observations is that in Refs. [20,26], the SOT is dominated by the dampinglike component, while in Eq. (14), the fieldlike torque dominates.

The charge pumping induced by a time-varying magnetization presents similar features as it is the Onsager reciprocal of the SOTs. Interestingly, the pumped charge current in turn enhances the magnetic damping of the in-plane magnetization components. Although the magnitude of the enhanced damping calculated in the present work is consistent with the experimental observations [24], one cannot rule out that other effects, such as the SHE of the TI bulk states [69], could also contribute to the spin-charge conversion process in these systems.

In conclusion, while the standard theory of magnetic TI surfaces derived in the present work can account for some of the features observed experimentally, some major discrepancies (in particular the angular dependence and the magnitude of the damping torque) cannot be explained. These limitations suggest that the coupling between the magnetic material and the TI surface [29,49], as well as the contribution of bulk states [69], should be taken into account to model the experiments.

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APPENDIX: INTEGRATION OF $\delta S^\|$ WHEN $\epsilon_F > \Delta$

In this appendix, we further clarify the significance of the Fermi-sea contribution to the nonequilibrium spin density $\delta S^\|$ given by Eq. (9). Let us demonstrate that this contribution vanishes in the metallic regime. Such a demonstration was carried out by Sinitsyn et al. [52] for the anomalous Hall
effect, and we now explicitly extend it to the nonequilibrium spin density. We can notice straightforwardly that $\delta S^{II}$ is, by construction, even in scattering time $1/\tau$ [Eq. (9)] involves only terms like $\sim \hat{G}^R \hat{G}^R$ and $\sim \hat{G}^A \hat{G}^A$. Therefore, in the limit of long relaxation time, $\delta S^{II} \approx \delta S^{II}_{int} + O(1/\tau^2)$, where $\delta S^{II}_{int}$ is the intrinsic contribution in the absence of disorder. The higher-order contributions lie beyond the scope of our study since we search only for terms $\sim \tau$ and $\sim 1 + O(1/\tau)$ [see Eq. (13)]. Hence, our aim is to demonstrate that the intrinsic contribution, $\delta S^{II}_{int}$, vanishes.

The strategy is to write down Eq. (9) in the chiral basis $\{|u^+\rangle, |u^-\rangle\}$ given in Eq. (6). In this basis, the retarded (advanced) Green’s function reads $\hat{G}^{R(A)} = \sum_{\tau} \langle u^{\tau} \mid u^{\tau}\rangle$. Let us now decompose the energy integral into positive and negative energy regions:

$$\delta S^{II}_{int} = \frac{\hbar}{2\pi} \text{Re} \int_{-\infty}^{0} d\varepsilon f(\varepsilon) \text{tr}\{ \cdots \}, \quad (A1)$$

$$\delta S^{II}_{int} = \frac{\hbar}{2\pi} \text{Re} \int_{0}^{+\infty} d\varepsilon f(\varepsilon) \text{tr}\{ \cdots \}. \quad (A2)$$

In the different regions, the unperturbed Green’s function reads

$$\hat{G}^{R(A)}_{\varepsilon < 0} = \frac{|u^+\rangle \langle u^+|}{\varepsilon - \varepsilon^+_k + i 0^+} + \frac{|u^-\rangle \langle u^-|}{\varepsilon - \varepsilon^-_k + i 0^+}, \quad (A3)$$

$$\hat{G}^{R(A)}_{\varepsilon > 0} = \frac{|u^+\rangle \langle u^+|}{\varepsilon - \varepsilon^+_k - i 0^+} + \frac{|u^-\rangle \langle u^-|}{\varepsilon - \varepsilon^-_k - i 0^+}. \quad (A4)$$

Then, Eqs. (A1) and (A2) can be rewritten as

$$\delta S^{II}_{int} = -\hbar \text{Im} \int \frac{d^2k}{(2\pi)^2} \frac{F^{+}_k}{(\varepsilon^+_k - \varepsilon^-_k)^2}, \quad (A7)$$

$$\delta S^{II+}_{int} = -\hbar \text{Im} \int \frac{d^2k}{(2\pi)^2} \frac{F^{+}_k}{(\varepsilon^+_k - \varepsilon^-_k)^2}, \quad (A8)$$

$$\delta S^{II+}_{int} = -\hbar \text{Im} \int \frac{d^2k}{(2\pi)^2} \frac{\delta(\varepsilon^+_k - \varepsilon^-_k)}{\varepsilon^+_k - \varepsilon^-_k}. \quad (A9)$$

In Eq. (A7), the integration over $k$ lies in the range $[0, +\infty)$, while in Eq. (A8) it runs over $[0, k_F]$, where $k_F$ is the solution of $\varepsilon^+_k = \varepsilon_F$ and depends on the angle $\phi_k$. Therefore, one can rewrite these expressions explicitly as

$$\delta S^{II+}_{int} = \frac{\hbar v}{2} \int_{0}^{2\pi} \int_{0}^{+\infty} d\phi_k dkd\varepsilon_k \frac{1}{(2\pi)^2} \frac{1}{\varepsilon^+_k} \{ \Delta[\cos \theta e\varepsilon - (m \cdot e\varepsilon)z] + \hbar v[(z \times k) \cdot e\varepsilon]z \}, \quad (A10)$$

$$\delta S^{II+}_{int} = -\frac{\hbar v}{2} \int_{0}^{2\pi} \int_{0}^{+\infty} d\phi_k dkd\varepsilon_k \frac{1}{(2\pi)^2} \frac{1}{\varepsilon^+_k} \{ \Delta[\cos \theta e\varepsilon - (m \cdot e\varepsilon)z] + \hbar v[(z \times k) \cdot e\varepsilon]z \}, \quad (A11)$$

$$\delta S^{II+}_{int} = \frac{\hbar v}{2\varepsilon_F} \int_{0}^{2\pi} \int_{0}^{+\infty} d\phi_k dkd\varepsilon_k \frac{1}{(2\pi)^2} \frac{1}{\varepsilon^+_k} \{ \Delta[\cos \theta e\varepsilon - (m \cdot e\varepsilon)z] + \hbar v[(z \times k) \cdot e\varepsilon]z \} \delta(\varepsilon_F - \varepsilon^+_k). \quad (A12)$$

Hence, it is sufficient to calculate $\delta S^{II+}_{int} + \delta S^{II-}_{int}$ and $\delta S^{II+}_{int}$. After some algebra, we get

$$\delta S^{II+}_{int} + \delta S^{II-}_{int} = \frac{\Delta}{4\pi} \frac{1}{\hbar v \varepsilon_F} \cos \theta e\varepsilon, \quad (A13)$$

$$\delta S^{II+}_{int} - \delta S^{II-}_{int} = -\frac{\Delta}{4\pi} \frac{1}{\hbar v \varepsilon_F} \cos \theta e\varepsilon, \quad (A14)$$

and then $\delta S^{II}_{int} = \delta S^{II+}_{int} + \delta S^{II-}_{int} + \delta S^{II+}_{int} = 0$. Consequently, the Fermi-sea contribution to the nonequilibrium electrically induced spin density vanishes in the metallic limit, within the weak-scattering regime.